PHYS 4A – Spring 2024 – Midterm 1

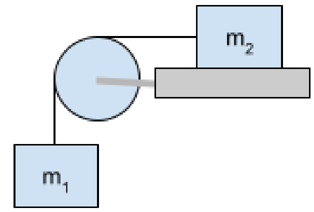
Question:	1	2	3	4	5	Total
Points:	6	8	8	8	0	30
Bonus Points:	0	0	0	0	5	5
Score:						

1. (6 points) For projectile motion starting and ending at the same height, the distance is given by the formula:
$$d = \frac{v_i^2 \sin 2\theta_i}{g}$$
where v_i is the initial *speed* and θ_i is the angle to the horizontal. A soccer player passes the ball to his teammate 20.0 meters away in a projectile motion. Given that he normally kicks the ball at 20.0 m/s, how long is the ball in the air?

2. (8 points) A woman is driving her car and her phone is lying flat on the dashboard. There is a curve coming up and the radius of the curve is 100 meters. Assuming the static friction coefficient between the phone and the dashboard is 0.800, what is the maximum speed she can have when entering into the curve so that the phone doesn't slide. Assume the car's speed before entering the curve and in the curve is constant.

3. (8 points) In transatlantic air travel between New York and London the flight time on an Airbus 380 from New York to London (eastwards) is 5h30 but the return journey (westwards) takes 6h30. The jetstream, which can be assumed to be a *constant wind* at the airplane's cruising altitude, creates this effect as the airplane flies in the jetstream. The airplane's normal cruising speed with respect to air is 920 km/h. Assuming that the airplane's trajectory and jetstream are completely aligned, calculate the *speed* and *direction* of the jetstream (east or west) as well as the *distance* between New York and London.

4. The friction coefficients between the table and m_2 are $\mu_s = 0.800$ and $\mu_k = 0.600$. The masses are $m_1 = 10.0\,\mathrm{kg}$ and $m_2 = 10.0\,\mathrm{kg}$. The pulley and the rope are massless.



   (a) (4 points) What is the acceleration of m_1?

   (b) (4 points) How would we have to change m_2 for the system to be *stationary*? Calculate the minimum value of m_2 that would achieve this.

5. (5 points (bonus)) A typical motorcycle has its engine connected *only* to the rear wheel. Assuming that 50% of the combined weight of the motorcycle and the rider is carried by the rear wheel, what is the maximum acceleration the motorcycle can attain if $\mu_s = 0.9$ and $\mu_k = 0.7$.

Differential Equations

$$\vec{\mathbf{v}} = \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t} \quad \text{velocity}$$
$$\vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\mathbf{r}}}{\mathrm{d}t^2} \quad \text{acceleration}$$
$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}(0) + \int_0^t \vec{\mathbf{v}}(t)\,\mathrm{d}t$$
$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}(0) + \int_0^t \vec{\mathbf{a}}(t)\,\mathrm{d}t$$

1-D Constant Acceleration

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$
$$x_f = x_i + \frac{1}{2}(v_i + v_f)t$$
$$v_f = v_i + at$$
$$v_{\text{avg}} = \frac{1}{2}\left(v_i + v_f\right)$$
$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Polar coordinates $(r, \theta) \to (x, y)$

$$\vec{\mathbf{r}} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = \begin{pmatrix} r\cos\omega t \\ r\sin\omega t \end{pmatrix}$$

2-D Constant Acceleration

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$
$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \frac{1}{2}(\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f)t$$
$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$$
$$\vec{\mathbf{v}}_{\text{avg}} = \frac{1}{2}\left(\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_{\mathbf{f}}\right)$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

Relative Motion

$$\vec{\mathbf{v}}_{C/A} = \vec{\mathbf{v}}_{C/B} + \vec{\mathbf{v}}_{B/A}$$
$$\vec{\mathbf{v}}_{B/A} = -\vec{\mathbf{v}}_{A/B}$$

Circular Motion with a constant radius

$$\vec{\mathbf{a}}_{\text{cent}} = -\omega^2 r\,\hat{\mathbf{r}} = -\frac{v_{\text{tang}}^2}{r}\,\hat{\mathbf{r}}$$
$$\vec{\mathbf{a}}_{\text{tang}} = \frac{\mathrm{d}v_{\text{tang}}}{\mathrm{d}t}\hat{\mathbf{v}}$$
$$\omega = \frac{2\pi}{T} \quad \text{when} \quad \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0$$
$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{v_{\text{tang}}}{r}$$

$\hat{\mathbf{v}}$ and $\hat{\mathbf{r}}$ unit vectors in the $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ directions respectively

$$1\,\text{rev} = 2\pi\,\text{rads}$$

Friction forces

$$F_{\text{static}} \leq \mu_{\text{static}} F_{\text{Normal}}$$
$$F_{\text{kinetic}} = \mu_{\text{kinetic}} F_{\text{Normal}}$$

Newton's Laws (inertial frame)

$$\sum_{\text{all}} \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Resistance proportional to v (in oil/water)[1]

$$\vec{\mathbf{R}} = -b\vec{\mathbf{v}} \quad \text{resistive force}$$
$$v_T = \frac{mg}{b} \quad \text{terminal velocity in free fall}$$
$$v(t) = v_T\left(1 - e^{-t/\tau}\right) \quad \text{with } \tau \equiv \frac{m}{b}$$

Resistance proportional to v^2 (air)[2]

$$\left|\vec{\mathbf{R}}\right| = \frac{1}{2}D\rho A v^2 \quad \text{resistive force}$$
$$v_T = \sqrt{\frac{2mg}{D\rho A}} \quad \text{terminal velocity in free fall}$$

Quadratic formula

$$\text{If} \quad 0 = ax^2 + bx + c$$
$$\text{Then} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[1] τ the *characteristic time* and b the proportionality constant.

[2] D a dimensionless constant, ρ the density of the medium in which the object is travelling, A the surface area of the object *seen from the direction of travel*