De Anza College

First Name ____ Campus ID ____

Last Name ____

PHYS 4A – Spring 2024 – Midterm 2

Question:	1	2	3	4	Total
Points:	6	8	8	8	30
Bonus Points:	2	2	0	0	4
Score:					

Remember to pay attention to significant figures and don't get lost in long calculations. The answers are usually only a few lines.

1. There is a truck filled with water with a combined mass of 5.00×10^3 kg and it's at rest. Suddenly the valve for the tank is opened and water starts flowing out horizontally at a rate of 5.00 kg/s with a speed of 20.0 m/s relative to the truck.

Hint: Use rocket equations

(a) (3 points) What is the thrust force created by the water jet on the truck?

Solution: The thrust force is given by the formula from the rocket equation

$$F_{\text{thrust}} = v_e \frac{\mathrm{d}m}{\mathrm{d}t} = 20.0\,\mathrm{m/s} \times 5.00\,\mathrm{kg/s} = 100\,\mathrm{N}$$

(b) (3 points) Assuming mass flow rate and speed is constant, what is the final velocity of the truck after 2.00×10^3 kg of water have been ejected?

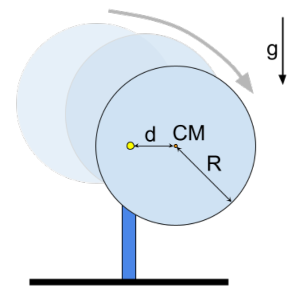
Solution: The equation for the rocket propulsion is used to calculate the change in velocity due to changing mass of the object where v_f is sought with $v_i = 0$, $M_f = 3.00 \times 10^4$ kg and $M_i = 5.00 \times 10^4$ kg

$$v_f - v_i = v_e \ln \frac{M_i}{M_f} = 20\,\mathrm{m/s} \times \ln 1.666 = 10.2\,\mathrm{m/s}$$

(c) (2 points (bonus)) Is the acceleration of the truck constant? If not, when is acceleration highest and when is it lowest?

Solution: The acceleration is not constant. As the thrust force is constant, acceleration is lowest when the truck is heaviest, i.e. full of water. And acceleration is highest when the truck is lightest, just as the water is running out.

2. A uniform disc of mass m and radius R is attached to a vertical pole with a frictionless hinge a distance d away from it's center and allowed to rotate freely subject to gravity. The **lowest** angular velocity observed is ω_L. *Ignore air friction.* $I_{\text{CM}} = mR^2/2$



(a) (2 points) At what positions will ω_L and ω_H be observed respectively? (Describe as the position of the *center of mass* CM with respect to the hinge).

Solution: The object will have the lowest angular velocity when the CM is directly above the hinge, and the highest angular velocity when the CM is directly below the hinge.

(b) (2 points) What is the difference in potential energy between the positions corresponding to ω_L and ω_H

Solution: The potential energy difference is $\Delta U_g = 2mgd$

(c) (4 points) Calculate the expression for ω_H in terms of d, R, g, ω_L.

Solution: Energy conservation between the highest and lowest angular velocity give us:

$$\frac{1}{2}I\omega_L^2 + 2mgd = \frac{1}{2}I\omega_H^2 \quad \Rightarrow \quad \omega_H = \sqrt{\omega_L^2 + \frac{4mgd}{I}}$$

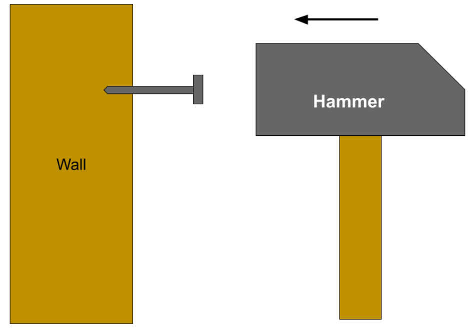
Using $I = mR^2/2 + md^2$ we get

$$\omega_H = \sqrt{\omega_L^2 + \frac{4gd}{R^2/2 + d^2}}$$

(d) (2 points (bonus)) Show that if $d = 0$ then $\omega_L = \omega_H$. Explain why that's the case.

Solution: When the object rotates around it's center of mass then it's potential energy will not change with rotation. Therefore it's angular velocity will be constant. We can see it from the second term in the above expression going to zero leaving only ω_L^2 inside the square root.

3. When hammering a nail into wood, the worker gives the hammer an initial speed of $v_i = 10.0\,\mathrm{m/s}$ just before it hits the nail. The hammer drives the nail in by $d = 1.50\,\mathrm{cm}$ and comes to a complete stop. The nail experiences a resistive force of $F_R = 1.20\,\mathrm{kN}$ to penetrate the wood.



(a) (2 points) What is the total work done on the nail?

Solution: Total work on the nail is $W = F_R \times \Delta x = 18.0\,\mathrm{J}$

(b) (2 points) Is momentum conserved for the hammer? Explain.

Solution: Momentum is not conserved for the hammer because the nail exerts a resistive force on the hammer. Momentum is only conserved when the total force is zero.

(c) (4 points) Calculate the mass of the hammer assuming all the energy is transferred to the nail.

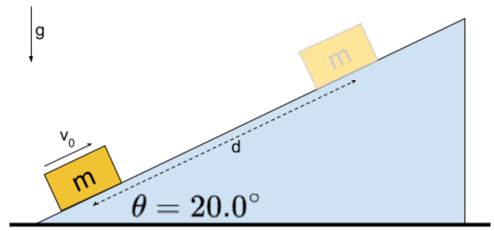
Solution: The hammer needs to have sufficient kinetic energy to drive the nail into the wood. Therefore:

$$\frac{1}{2}mv^2 = 18.0\,\mathrm{J}$$

using the velocity of the hammer at $10.0\,\mathrm{m/s}$ we get:

$$m = \frac{2 \times 18.0\,\mathrm{J}}{(10.0\,\mathrm{m/s})^2} = 360\,\mathrm{g}$$

4. An object of mass $m = 1.00\,\text{kg}$ is given an initial velocity of $v_0 = 8.00\,\text{m/s}$ on an incline with an angle $\theta = 20.0°$. The object comes to rest after travelling a distance $d = 7.00\,\text{m}$ on the incline due to friction.



(a) (2 points) Draw a box within which energy is conserved and calculate the initial energy of the system.

Solution: Energy is conserved for the system including the incline and the object. The initial energy of the system would be:

$$E_i = \frac{1}{2}mv_i^2 = 32.0\,\text{J}$$

(b) (2 points) Calculate the amount of non-conservative work done.

Solution: The final energy of the system is

$$E_f = E_{\text{int}} + mgd\sin\theta$$

But as $E_i = E_f$:

$$18.0\,\text{J} = E_{\text{int}} + 1.00\,\text{kg} \times 9.81\,\text{m/s}^2 \times 7.00\,\text{m} \times \sin 20.0°$$

Hence $\Delta W_{\text{non-cons}} = E_{\text{int}} = 8.51\,\text{J}$

(c) (4 points) Calculate the coefficient of friction μ_k.

Solution: The normal force is $F_N = mg\cos\theta$ and $F_k = \mu_k F_N$. Therefore $W_{\text{non-cons}} = F_k d$ results in:

$$\mu_k mgd\cos\theta = W_{\text{non-cons}} \quad\Rightarrow\quad \mu_k = \frac{W_{\text{non-cons}}}{mgd\cos\theta}$$

Plugging in the numbers we get $\mu_k = 0.132$.